AN IMPROVED SPECTRAL WIDTH DOPPLER METHOD FOR ESTIMATING DOPPLER ANGLES IN FLOWS WITH EXISTENCE OF VELOCITY GRADIENTS

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(Received 28 May 2005, revised 27 April 2006, in final form 4 May 2006)

Abstract—Doppler angle (i.e., beam-to-flow angle) is an important parameter for quantitative flow measurements. With known Doppler angles, volumetric flows can be obtained by the mean flow velocity times the cross-section area of the vessel. The differences or changes between prestenotic and poststenotic volumetric flows have been quantified as an indicator for assessing the clinical severity of the stenosis. Therefore, several research groups have dedicated themselves to developing user-independent methods to determine automatically the Doppler angle. Nevertheless, most of these methods were developed for narrow ultrasound beam measurements. For small vessels, where the beam width is a significant fraction of the diameter of the vessel, the effect of velocity gradients plays an important role and should not be ignored in the Doppler angle estimations. Accordingly, this paper is concerned with a method for improving the estimation of Doppler angles from spectral width Doppler (SWD) method, but correcting for velocity-gradient broadening that may arise when the beam has a nonzero width. In our method, Doppler angles were firstly calculated by SWD and then were corrected by an artificial neural network (ANN) method to neutralize the contribution of velocity gradient broadening (VGB). This SWD and ANN conjoint method has been successfully applied to estimate Doppler angles from 50° to 80° for constant flows in 10 mm, 4 mm and 1 mm diameter tubes, whose mean flow velocities were 15.3, 19.9 and 25.5 cm/s, respectively, and the achieved mean absolute errors of the estimated Doppler angles were 1.46°, 1.01° and 1.3°.

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Key Words: Doppler ultrasound, Spectral-width Doppler, Velocity-gradient broadening, Artificial neural networks.

INTRODUCTION

Ultrasound is the most widely used imaging modality for detecting abnormalities in peripheral vessels. With the combination of high-resolution gray-scale ultrasound tissue image and Doppler flow information, ultrasound permits a noninvasive way for displaying hemodynamic distributions of superficial vessels with high sensitivity. Unlike contrast angiography, studies using ultrasound can be repeated longitudinally to follow the natural history of disease in a cost-efficient way (Zarins et al. 1983). In flow velocity measurements using Doppler ultrasound, the component of the flow velocity parallel to beam axis can be measured by the utilization of the Doppler shift principle. The true flow velocity can then be calculated (eqn (A1)) if the Doppler angle (i.e., the angle between the ultrasound beam and the direction of blood flow) is known. The differences between the prestenotic and poststenotic volumetric flows have been subsequently quantified as an indicator for assessing the clinical severity of the stenosis (Bluth et al. 1988; Robinson et al. 1988). In clinical Doppler ultrasound, the most common procedure for obtaining the requisite Doppler angle is done by manually aligning the vessel axes on two-dimensional (2-D) ultrasound images (Ho et al. 2002). The accuracy of this manual process usually depends on
Moreover, in some cases (e.g., eccentric stenosis), the flow direction of a poststenotic jet depends on the morphology of the stenosis whose flow directions might be unparallel to the blood vessel axes, and their Doppler angles will not be able to be estimated from the aforementioned manual process. Similarly, other complex flow patterns, such as helices, recirculation zones, turbulent flows, etc., having their flow directions complicated, also make the manual process ineffectual in Doppler angle detections (Dunmire et al. 2000; Khoshnian et al. 2005).

To estimate the Doppler angles accurately, a number of user-independent approaches have been proposed by measuring the velocity vectors perpendicular and parallel to the beam axis (Dunmire et al. 2000; Lee et al. 1999a; Newhouse et al. 1987; Yeh and Li 2002). One approach is the speckle tracking method. The 2-D velocity vectors were detected by tracking the speckle patterns from consecutive B-mode images (Bohs et al. 1995, 1998). Another approach is the cross-beam vector Doppler systems. One or multiple ultrasound beams are organized in one-dimensional (1-D) or 2-D set-up to measure velocity vectors of blood flows that are perpendicular and parallel to ultrasound beam axes (Dunmire et al. 2000). Dunmire (1998) constructed a vector Doppler system with one ultrasonic transmitter and two receivers. The Doppler signals from two incident angles were utilized to resolve the 2-D velocity vectors on the scanning plane. Capineri et al. (2002) implemented a linear array system. Two-dimensional flow maps were constructed from measured 2-D velocity vectors by applying ultrasound beams in multiple incident angles. Hein (1995) further extended the transducer set-up from 1-D to 2-D and designed a three-transducer system to measure the three-dimensional (3-D) velocity vectors. Besides, Newhouse et al. (1987) proposed a transverse Doppler method and found that the velocity vectors perpendicular to the ultrasound beam can be estimated by measuring the spectral bandwidth on the measured Doppler spectra. The implication between the spectral bandwidth and velocity vector lateral to ultrasound beam is based on the fact that Doppler signals generated from scatterers passing through a focused ultrasound beam are influenced by both the limited lateral extent of a focused ultrasound beam and the presence of several local insonation angles. Guidi et al. (2000) further studied the spectral broadening effect and found that the effect can be formulated as a combination of transit-time broadening and geometrical spectral broadening; the combination of these two effects was referred to as the intrinsic spectral broadening (ISB) effect (Guidi et al. 2000). With the combination of classical Doppler and transverse Doppler, flow velocities parallel and perpendicular to ultrasound beam axis, as well as Doppler angles, can be measured. Newhouse et al. (1994) also proposed a two-ultrasound beam approach for 3-D velocity vector measurements (Newhouse et al. 1994). Lee et al. (1999a, 1999b) took the advantage of axial symmetry of annular array and proposed a spectral width Doppler (SWD) method for Doppler angle estimation by combining classical Doppler and the transverse Doppler. One noticeable advantage of the transverse Doppler is that it provides additional information of the lateral flow velocity from spectral bandwidth and, therefore, the system complexity for Doppler angle measurements can be reduced. These transverse Doppler-based vector Doppler systems can measure 2-D velocity vector by 1-D transducer set-up and 3-D velocity vector by 2-D transducer set-up. The aim of this paper is to propose an improved method based on this spectral width Doppler system for estimating Doppler angles in flows where there are presences of velocity gradients.

Because SWD systems intend to estimate the velocity vectors normal to an ultrasound beam from the measured spectral bandwidth, a narrow ultrasound beam and an ultrashort ultrasonic burst are always required. However, implementation of an ultrasound beam with extremely narrow beam width is impractical, especially in small vessel measurements. An ultrasound beam with nonzero (nonideal) beam width will incur the inclusion of velocity gradients within the sample volume, which makes the measured Doppler spectra within the sample volume composed of flow-line spectra generated from multiple velocities. Specifically, for small vessels, where the beam width is a significant fraction of the diameter, there may be velocity-gradient broadening (VGB) within the sample volume (Denardo et al. 1994). Therefore, one concern when applying SWD systems to flows with the presences of velocity gradients is that the measured spectral bandwidth is actually a mixture of the ISB with velocity-gradient broadening (VGB). The mixture of VGB and ISB in the measured spectral width will deteriorate the mapping relation between ISB and spectral width and will inevitably cause poor performance, when applying SWD system for small vessel measurements. Accordingly, the effect of velocity gradients is a parameter which should be taken into consideration in SWD measurements.

The effect of VGB can be studied with respect to velocity gradients by separating the flows in a sample volume into several flow lines. Considering a sample volume with a cylindrical shape, the sample volume can be spatially defined by the beamwidth, $R_s$, and the sample volume length (height), $L_o$, on cylindrical coordinates.
sound beamwidth divided by vessel diameter (beam-width-to-vessel-diameter ratio (BWR) as ultra-vessel wall (Fei 1995). Fei (1995) further defined the cause the effective flow lines are confined within the expressed as a function of ultrasound beam width be-

Furthermore, if the sample volume length is set longer than the length of the intersection area (i.e., $L_s = R_v / \sin \theta$) and the flow is axially symmetric with the central axis, the flow lines inside a sample volume can be simply expressed as a function of ultrasound beam width because the effective flow lines are confined within the vessel wall (Fei 1995). Fei (1995) further defined the beam-width-to-vessel-diameter ratio (BWR) as ultrasound beamwidth divided by vessel diameter ($R_v / R_s$) to facilitate the modeling of the effect of the VGB on measured Doppler spectra. A BWR value of “0” represents an extremely thin ultrasound beam, while a BWR value of “1” means a wide ultrasound beam with its width equivalent to the vessel diameter. The sample volume covers greater velocity gradients along with the increase of BWR value. For conditions of BWR values greater than “1”, the vessel is fully insonated by the ultrasound beam and the flow lines inside the vessel are wholly included. The relationship between the BWR and VGB on the measured Doppler spectra has been discussed in several articles (Baker et al. 1978; Evans 1982, 1985; Gill 1985; Oates et al. 1990). Bascom et al. (1986, 1990) elucidated the relationship between Gaussian ultrasound beam and Doppler spectra by means of numerical simulation. Kagiymama et al. (1999) measured Doppler spectra with different BWR values and proposed that the BWR value should be smaller than 0.5 for accurate volumetric flow measurements. Fei (1995) proposed a theoretical simulation model to illustrate the relationship between the measured volumetric flows and BWR values. He also proposed a correction model for volumetric flows based on BWR values. Nevertheless, these papers did not discuss the contribution of ISB (Guidi et al. 1995) and nonzero beam width model (Fei 1995), Doppler spectra can be simulated if the flow profile is known. The contribution of the VGB in the SSWD-estimated angle can be numerically calculated and corrected by applying a well-trained ANN. The relationship between the “uncorrected” and “corrected” SSWD angles can also be numerically calculated under different BWR conditions. In this study, back-propagation neural network (BPNN) was chosen to model the relationship between the “uncorrected” and “corrected” SSWD angles in a wide range ($45^\circ \leq \theta \leq 85^\circ$) and the BWR values were varied from 0 to 1 ($0 \leq \text{BWR} \leq 1$). The proposed SSWD and BPNN conjoint method has been successfully implemented to estimate Doppler angles from $50^\circ$ to $80^\circ$ in 10-mm, 4-mm and 1-mm-diameter tubes.

MATERIALS AND METHODS

Doppler system

In our study, Doppler spectra were measured by a color Doppler ultrasound system (GE VingMed, CFM-750) equipped with a four-element annular array transducer, driving at 4 MHz. The transducer is 1.47 cm in diameter and 7.8 cm in focal length. The transducer produces a 2-mm beam width defined by $-6$-dB lateral width at focus (Fig. 1). The pulse repetition frequency (PRF) was set at 4 kHz and complex discrete Fourier transform (complex DFT) was applied every 1ms to obtain the Doppler spectra. The Doppler spectra obtained

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**Fig. 1.** The schematic diagram of the set-up for Doppler angle measurements. The $R_v$, $L_v$, $R_s$, $L_s$, $\theta$, $v$, $F$ and $W$ represent the ultrasound beamwidth, sample volume length, vessel diameter, vessel length, Doppler angle, flow velocity, transducer focal length and diameter of transducer, respectively. The $(x, y, z)$ represents the vessel coordinate system and $(x', y', z')$ is the transducer coordinate system.
Laminar flows were generated using a UHDC flow phantom under constant flow conditions (Shelley Medical Imaging Technologies, London, Ontario, Canada), whose volumetric output can be controlled by a programmable microprocessor-controlled piston with finest resolution of 0.01 cm³/s. UHDC blood mimic fluid was chosen as the scattering source and had viscosity of 1.34 cp (1 cp = 10⁻² g/s · cm), density of 1.01 g/mL, and 1540 m/s sound speed at 20°C. Polyethylene tubes with 10, 4 and 1 mm diameters were used in our studies, which corresponded to BWR values of 0.2, 0.5 and 2, respectively. Three constant flows were generated in the tubes corresponding Reynolds numbers were 1042, 543 and 1 mm diameters were used in our studies, which corresponded to BWR values of 0.2, 0.5 and 2, respectively. Three constant flows were generated in the tubes with mean flow velocities of 15.3, 19.9 and 25.5 cm/s, respectively. The mean flow velocities in the polyethylene tubes were calculated as the piston’s flow volume output divided by the vessel’s cross-section area. Their corresponding Reynolds numbers were 1042, 543 and 174 in 10, 4 and 1 mm, respectively.

Doppler spectrum simulation using summation of flow-line spectra

The Doppler spectra were simulated using the technique of flow-line spectrum summation, which assumes that the Doppler spectrum can be represented as a linear summation of flow-line spectra in the sample volume (Guidi et al. 1995; Bharath and Kitney 1992). The sample volume was simulated in a cylindrical shape (Brown et al. 1985) with sample volume length, L_s, and ultrasound beam width, R_v. The scatterers were confined in a round tube which has length L_v and diameter R_v. Scatterers in the tube were simulated under laminar-flow condition with interspace of 1/1000 · R_v distributed in x, y and z directions.

\[ x' = x \]
\[ y' = -\left(y'\cos \theta + z'\sin \theta\right) \]
\[ z' = y'\sin \theta - z'\cos \theta \]

where \( \theta \) is the Doppler angle.

For Doppler ultrasound simulation, only the scatterers inside the tube were defined (i.e., \( x < \sqrt{\frac{R_v^2}{2}} - y^2 \), \( -\frac{R_v}{2} < y < \frac{R_v}{2} \), \( -\frac{L_v}{2} < z < \frac{L_v}{2} \)). These scatterers defined in terms of the tube coordinate can be transformed into transducer coordinate within the ranges of \( x < \sqrt{\frac{R_v^2}{2}} - (y'\cos \theta + z'\sin \theta)^2 \), \( y' < (R_v\cos \theta + L_v\sin \theta) \), and \( z' < (R_v\sin \theta + L_v\cos \theta) \). In the sample volume, the summed spectra, \( P_{sv}(f) \), can be represented as
where $P_{sv}(f)$ represents the summed Doppler spectrum, $L_v$ and $R_v$ are the length and the diameter of the tube, $P(f, x', y', z')$ is the flow-line spectrum at $(x', y', z')$, $b(x', y')$ is the 2-D Gaussian beam shape function in $x'y'$-plane with $-6$ dB later width equaled to $R_v$, $\Phi(x', y', z')$ is the selection function for defining the scatterers inside the sample volume, $L_s$ is the sample volume length and $R_s$ is the diameter of ultrasound beam width.

The simulated Doppler spectrum was then normalized to its peak, which can be represented as

$$P_{sv,n}(f) = P_{sv}(f)/P_{sv}(f_d),$$

(3)

where $P_{sv,n}(f)$ is the normalized Doppler spectrum and $f_d$ is the energy peak frequency in the simulated Doppler spectrum.

Figure 3 shows the comparison between the measured and simulated Doppler spectra, whose frequency axes have been frequency-normalized to their peak frequency, $f_d$. Their vertical axes representing the intensities of the spectra are normalized to their peak values separately. The Doppler spectra in Fig. 3a and b show the Doppler spectra obtained in 4-mm and 1-mm-diameter tubes, respectively. The mean flow velocities in the 4-mm and 1-mm tubes were 19.9 and 25.5 cm/s for constant flows simulated or measured at 60° Doppler angle. The sample volume length was set at 8 mm to fully cover the intersectional region of the tubes with the ultrasound beam. In Fig. 3a and b, the upper frequency portions ($f_{max}/f_d \geq 1$) of the simulated spectra are well matched with the measured spectra, whereas their lower frequency portions are not ($f_{max}/f_d < 1$). The discrepancy of the lower frequency portions in Fig. 3a and b could be owing to the contaminations of the Doppler signals from slow aggregated particles (Paeng et al. 2001), low-frequency noise and influence of wall-thump filter (Lee et al. 1999a). However, the $-3$-dB thresholded $f_{max}/f_d$ ratios in the measured and simulated Doppler spectra are quite similar, which are 1.28 and 1.3 in the 4-mm tube and 1.31 and 1.33 in the 1-mm tube. In our study, the SWD method only utilizes the information of $f_{max}/f_d$ (see below) for Doppler angle estimations, so that the presence
Measurement procedures

A symmetrically focused ultrasound beam was generated by an annular array transducer to measure the backscattered Doppler signals. The ultrasound probe was clamped in a probe carriage device. Tilts of the ultrasound probe were controlled by a microprocessor-controlled mechanical step motor with 0.1° resolution. Cross-section views of vessels were acquired by B-mode ultrasound to monitor the sample volume position so that the sample volume can be positioned at the center of the tube. A protractor was used to double-check the correctness of the Doppler angles. To standardize the contributions of the sample volume length in the measured Doppler spectrum, the sample volume lengths were set longer than the intersectional regions of the sample volumes with the tubes. Doppler spectra were measured from 50° to 80° Doppler angles, \( \theta \), in 5° increment. Three thousand Doppler spectra were measured at each Doppler angle. Every 500 Doppler spectra were averaged for obtaining a noise-suppressed Doppler spectrum. These noise-suppressed Doppler spectra were then used to estimate Doppler angles using our SWD and BPNN conjoint method.

Estimation of Doppler angles in flows with velocity gradients using the SWD and BPNN conjoint method

SWD method (Lee et al. 1999a, 1999b) represents Doppler angles in terms of \( f_{\text{max}}/f_d \) as \( \theta = \tan^{-1}\left[ (f_{\text{max}}/f_d - 1) / (2FW) \right] \) (see Appendix A), where \( F \) is the focal length, \( W \) is the diameter of the annular array transducer and \( f_{\text{max}} \) and \( f_d \) are the maximum frequency and energy peak frequencies in the Doppler spectrum, respectively, when a focused ultrasound beam is used. The most widely used approach for \( f_{\text{max}} \) determination is to set a fixed threshold setting, \( \gamma_{th} \), (a fraction of the peak value in the power spectrum) to find the maximum frequency, \( f_{\text{max}} \), corresponding to that defined by \( \gamma_{th} \) (Tortoli et al. 1995; Chabria and Newhouse 1997). According to the theoretical analysis done by Newhouse et al. (1987), the theoretical frequency of the \( f_{\text{max}} \) in a measured Doppler spectrum can be approximated by setting a \( \gamma_{th} \) threshold as low as –20 dB. However, in real applications, the presence of noise in various forms may produce a noise floor that needs to be cut out by increasing the threshold level, which makes the theoretical value of \( f_{\text{max}} \) determined at –20 dB \( \gamma_{th} \) difficult to obtain (Guidi et al. 2000). In addition, the use of SWD for angle estimation makes an assumption of no presence of velocity gradients within the sample volume, which is impractical in small vessel measurements.

In this study, we develop an improved SWD method for velocity-gradient flows that can be realized at high \( \gamma_{th} \) level. Because the contribution of velocity gradients on spectral width can be calculated using numerical simulation, the effect of ISB can be segregated from the VGB in the Doppler spectra if the BWR value is given. To take the effect of VGB away from SWD estimated angles, one effective way is to model the relationship between the uncorrected SWD angles and the actual Doppler angles under different BWR values. Figure 4 shows the relationship when the \( \gamma_{th} \) level was set at –3 dB. It is noteworthy that the mapping from the actual Doppler angles to the uncorrected SWD angles at any BWR value has the characteristics of one-one mapping and piece-wise smoothing. Although this mapping is one-one mapping, we can observe in Fig. 4 that the mapping relationship is nonlinear. Taking the advantage of nonlinear modeling of ANN, we adopted back-propagation neural

![Fig. 4. The one-one mapping relationship between the actual and the uncorrected SWD angles. The nonlinear one-one relationship between the uncorrected and the actual Doppler angles under different BWR values.](image)

![Fig. 5. The architecture of the BPNN. The BPNN has two input neurons, 10 hidden neurons and one output neuron.](image)
The backpropagation neural network (BPNN) was used to model the nonlinear relationship. The BPNN has two input neurons, 10 hidden neurons and one output neuron (Fig. 5). The BPNN output is the linear summation of connection-weight values mapping to a sigmoid function \( f(x) = \frac{1}{1 + e^{-x}} \) (Newey and Nassiri 2002). The BWR value and the uncorrected SWD Doppler angle were used as the BPNN input, and their corresponding actual Doppler angles were treated as training targets. The training data for the BPNN should contain information comprehensive enough to represent the relationship between the uncorrected SWD angles and actual Doppler angles at a defined level.

The connection-weights before BPNN training were initially generated from a random function with values within –1 and 1. The learning rate of the BPNN was set at 0.1. Leave-one-out cross-validation (CV) was used as the early stopping criterion for BPNN training, to obtain the optimal setting of the hidden neuron number (Prechelt 1998). Due to the fact that signal quality of the training data poses a great influence on the performance of the BPNN, the training data were provided from simulation data (Doppler angles ranged from 45° to 85°; \( \gamma_{th} \) set at –2 dB, –3 dB and –6 dB; BWR values varied from 0.1 to 1). In this study, we created three BPNNs at –2 dB, –3 dB and –6 dB \( \gamma_{th} \) levels for Doppler angle corrections. The flowchart of the proposed SWD and BPNN conjoint method is shown in Fig. 6.

**RESULTS**

With the aid of Doppler spectrum simulation using flow-line spectrum summation (Guidi et al. 1995), Doppler spectra can be simulated at any Doppler angle, any

![Diagram](https://example.com/diagram.png)

**Fig. 6.** Flow chart of the proposed method for Doppler angle estimation.

![Diagram](https://example.com/diagram.png)

**Fig. 7.** The \( f_{max} \) and \( f_d \) in Doppler spectrum are insensitive to different sample volume lengths. (a) Intensity-normalized Doppler spectra, \( P_{nv}(f) \), were measured at 70° Doppler angle in 4-mm-diameter tube (BWR = 0.5, mean flow velocity = 19.89 cm/s) with sample volume lengths varied from 2 to 6 mm, and (b) their corresponding numerical simulated spectra.
sample volume size and any vessel size. Figure 7 presents an example of the measured and simulated Doppler spectra at 70° Doppler angle in a 4 mm diameter tube. The sample volumes were positioned at the central axis of the tube, and sample volume lengths were adjusted from 2 to 6 mm, to elucidate the effect of sample volume length on $f_{\text{max}}$ and $f_d$. The frequencies of −3 dB thresholded $f_{\text{max}}$ and $f_d$ were measured. The −3 dB thresholded $f_{\text{max}}$ were 649, 645, 643, 639 and 639 Hz when the sample volume lengths were adjusted from 2 to 6 mm and their $f_d$ were 543, 541, 540, 539 and 539 Hz, respectively. In Fig. 7a, the $f_{\text{max}}/f_d$ ratios were 1.2, 1.21, 1.19, 1.18 and 1.18 for 2-, 3-, 4-, 5- and 6-mm sample volume lengths. Compared with the simulated Doppler spectra in Fig. 7b, the $f_{\text{max}}/f_d$ ratios were between 1.34 and 1.22. This example illustrates that the $f_{\text{max}}/f_d$ ratio is insensitive to the sample volume length when the center of the sample volume is positioned at the central axis of the tube. It also can be found that nearly constant $f_{\text{max}}$ and $f_d$ were observed in the Doppler spectra with different sample volume lengths. The major differences among these spectra with different sample volume lengths were their lower frequency portions ($f/f_d < 1$). The spectral power in the lower frequency portion was raised with increase of sample volume length, both in the measured Doppler spectra (Fig. 7a) and in the simulated ones (Fig. 7b). This is owing to the inclusion of low-velocity flow lines near the vessel wall when a longer sample volume length is used. It is in line with the observation in previous studies conducted by Guidi et al. (1995) and

Fig. 8. The influence of BWR values on $f_{\text{max}}/f_d$ ratios. Doppler spectra of a constant flow with 19.9 cm/s mean velocity in a 4-mm-diameter tube were simulated. (a) The Doppler spectra were simulated at 50°, 60°, 70° and 80°, whose BWR values were varied from 0.1 to 1. (b) Normalizing the frequencies of the spectra in (a) to their $f_d$. (c) The frequencies of −3 dB thresholded $f_{\text{max}}$ and $f_d$. (d) The frequencies of $f_{\text{max}}$ and $f_d$ at 70° in (c).
Tortoli et al. (1995). In SWD systems, we only utilize the ratios of $f_{\text{max}}/f_d$ in Doppler angle estimations. Therefore, the effect of sample volume length is not significant in our SWD computation, because the ratio of $f_{\text{max}}/f_d$ is not likely to be affected by the lower frequency portion in a Doppler spectrum. Besides, the higher frequency portions in Fig. 3 and Fig. 7 show very good resemblance between the measured and simulated Doppler spectra, ...
not only in their $f_{\text{max}}/f_d$ ratios but also their spectra shapes. Because SWD systems only utilize the ratio of $f_{\text{max}}/f_d$ for estimating Doppler angles, the resemblances in spectral shape and $f_{\text{max}}/f_d$ ratio between the measured and simulated Doppler spectra demonstrate the feasibility of using flow-line spectrum summation as an approach in Doppler ultrasound studies.

To better understand how $f_{\text{max}}$ and $f_d$ are affected by BWR, Doppler spectra of a constant flow with 19.9 cm/s mean velocity in a 4-mm diameter tube were simulated, shown in Fig. 8. Figure 8a shows the Doppler spectra simulated at 50°, 60°, 70° and 80°, whose BWR values are ranged from 0.1 to 1. Owing to the inclusion of low-velocity flow lines in large BWR conditions, it can be observed that both the $f_{\text{max}}$ and $f_d$ had a declining tendency with the increase of BWR values. After normalizing the frequencies of these spectra to their $f_d$, the influence of BWR on $f_{\text{max}}/f_d$ can be seen in Fig. 8b. In Fig. 8a and b, the $f_{\text{max}}$ were determined at $-2$, $-3$ and $-6$ dB. The threshold levels, $\gamma_{th}$, were marked by dashed lines. The frequencies of $-3$ dB thresholded $f_{\text{max}}$ and $f_d$ were plotted in Fig. 8c, ranging from 50° to 80° Doppler angles with BWR values varied from 0.1 to 1. The $f_{\text{max}}$ and $f_d$ at 70° Doppler angle in Fig. 8c were plotted in Fig. 8d. It can be observed that the frequencies of $f_{\text{max}}$ and $f_d$ have tendencies of rapid decrease if BWR values are lower than 0.5. One noticeable point between the frequencies of $f_{\text{max}}$ and $f_d$ in Fig. 8d is the difference between their declining rates. The discrepancy in declining rates makes the ratio of $f_{\text{max}}/f_d$ decline fast when the BWR value changes from 0.2 to 0.6 and to approach to a value of 1.6 when the BWR value increases from 0.7 to 1.0.

A BPNN was applied to model the nonlinear relationship between the actual and the uncoupled SWD angles. The number of optimal hidden neurons of the BPNN was determined by leave-one-out cross-validation (CV) method (Prechelt 1998). Figure 9 shows the mean absolute error (MAE) of the cross-validation results (see Appendix B) when the BPNN with one to 20 hidden neurons were applied to a set of uncorrected SWD angles, which were simulated from 45° to 85° with 0.1 to 1.0 BWR values. The MAEs between the corrected and actual Doppler angles were between 0.79° and 1.6°. The detailed results of the corrected Doppler angles are listed in Table 1.

**DISCUSSION AND CONCLUSIONS**

This paper aims to develop a SWD-based angle estimation method that takes the effects of velocity gradients into account. This paper is based on the idea of SWD (Lee et al. 1999a, 1999b; Newhouse et al. 1987; Tortoli et al. 1995). The frequencies of $f_{\text{max}}$ and $f_d$ in the measured Doppler spectrum are bundled with the value of BWR for Doppler angle estimations. Although several methodologies have been reported for Doppler angle estimations, one distinct feature of the proposed method is its availability of being applied in velocity-gradient flows, which is usually induced from both the nonzero beam width and long-sample volume length. In this study, we discuss the influence of sample volume size on the Doppler spectral shape as well as its $f_{\text{max}}$ and $f_d$. We extended the study by Tortoli et al. (1995), which demonstrated the $f_{\text{max}}$ and $f_d$ are not sensitive to sample volume lengths in vessels whose diameters are much larger than the ultrasound beamwidth, and further studied the effects of BWR values on $f_{\text{max}}$ and $f_d$. This is the issue that was less discussed in previous papers about Doppler angle estimations.

The proposed method utilizes the information of $f_{\text{max}}$ and $f_d$ for Doppler angle estimation. It has the benefit of being less affected by the low-frequency noise in the Doppler spectra. Figure 3a and b show the Doppler spectra.
spectra at 60° Doppler angle obtained from the 4-mm and 1-mm-diameter tubes, respectively. The upper frequency portions, i.e., \( f/f_d > 1 \), in the measured Doppler spectra were well matched with the numerically simulated spectra. However, their lower frequency portions, i.e., \( 0 < f/f_d < 1 \), were not matched very well. This might be owing to the influences of cutoff frequency of the wall-thump filter, aggregated objects (Paeng et al. 2001) and low-frequency noise (Lee et al. 1999a, 1999b), which are usually seen in common ultrasound Doppler measurements. Because the lower-frequency portion in a Doppler spectrum is not used in our SWD computations (see eqn (A4)), it plays no influence in our Doppler angle estimations.

Figure 8 shows how the BWR affects the \( f_{\text{max}} \) and \( f_d \) in Doppler spectra. The \( f_{\text{max}} \) in a Doppler spectrum can be interpreted as the frequency corresponding to the “maximum” or axial velocity in the vessel. Similarly, the \( f_d \) can be interpreted as the Doppler frequency corresponding to the velocity that most scatterers inside the sample volume belong to. Owing to the inclusion of low-velocity flow lines near the vessel wall, both the values of \( f_{\text{max}} \) and \( f_d \) have declining tendencies with the increase of BWR value. Besides, as mentioned above...
(see Results), the $f_{\text{max}}$ has a faster decline rate than the $f_d$. The declining tendencies originate from the change of flow velocities existing in the sample volume when the value of BWR increases. For example, if we consider a Doppler measurement at 90° Doppler angle by applying a Gaussian ultrasound beam to measure laminar flow in a small vessel whose beamwidth is moderately wide between zero beamwidth and vessel diameter (see Fig. 11), the sample volume will cover the central axis ($V_{\text{max}}$) and parts of the outer rim ($v = 0$). Specifically, when the beam width is equal to or larger than the vessel diameter, the whole outer rim ($v = 0$) will be covered and, therefore, more low-velocity flow lines will be included in the sample volume. Because the energy peak frequency, $f_d$, corresponds to the majority of the flow velocities in the measured Doppler spectrum, it is expected that $f_d$ declines when the BWR increases. On the other hand, because the $f_{\text{max}}$ corresponds to the high-velocity flow lines near the central axis in the vessel, it should be insensitive to the change of BWR (Tortoli et al. 1995). However, to avoid being affected by noise floor (see Materials and Methods), we determine the $f_{\text{max}}$ in a relatively high threshold level, $\gamma_{th}$ (e.g., –3 dB), instead of using a low threshold value (e.g., ~20 dB), in this study. When the BWR goes higher, the involvement of low-velocity flow lines in the sample volume will increase the low-velocity scatterers and make the proportion of the composition of high-velocity scatterers decrease. In other words, if a fixed $\gamma_{th}$ is used, the read-out of $f_{\text{max}}$ will be underestimated in large BWR conditions. The declining tendencies of $f_{\text{max}}$ and $f_d$ with different BWR values were studied with the aid of numerical simulation. Once the relationship between the VGB and Doppler angles under different BWR values were calculated, the relationship between the actual Doppler angles and uncorrected SWD angles can be constructed. One example of the relationship between the actual Doppler angles and uncorrected SWD angles at –3 dB $\gamma_{th}$ is shown in Fig. 4.

Figure 8b illustrates the influences of BWR on $f_{\text{max}}/f_d$. The ratio of $f_{\text{max}}/f_d$ can be determined by setting a user-defined threshold level, $\gamma_{th}$. It can be seen in Fig. 8b that the $f_{\text{max}}/f_d$ ratio increases with the increment of BWR. This is owing to the discrepancy between the declining rates of $f_{\text{max}}$ and $f_d$. Because SWD was derived under the assumption of no presence of velocity gradients, it is expected that SWD will overestimate the Doppler angle under large BWR value conditions. Clearer illustration is shown in Fig. 8d. If we choose –3 dB $\gamma_{th}$, the $f_{\text{max}}/f_d$ ratios in Fig. 8d are 1.23, 1.25, 1.35, 1.47, 1.50, 1.56, 1.57, 1.58, 1.60 and 1.60 for BWR values from 0.1 to 1.0. The $f_{\text{max}}/f_d$ had a significant rise when the BWR value was increased from 0.2 to 0.5. Similar observations were also found in the ratios of $f_{\text{max}}/f_d$ at other Doppler angle conditions. These overestimated SWD angles are corrected by BPNN in this study.

The reason to choose an ANN for modeling the relationship between the actual and the uncorrected SWD angles is because of its ability to represent both linear and nonlinear relationships. Besides, neural networks also have the advantages of easy implementation and self-learning. Traditional linear mapping techniques are not suitable for modeling the nonlinear relationship. We have also used a third-order polynomial function to model the relationship in Fig. 4; the MAE between the actual Doppler angles and this polynomial model’s corrected angles was 3.71°, which was much larger than the result (MAE = 0.89°) corrected by BPNN (Table 1).
A good training dataset is prerequisite for modeling the relationship between the actual and the uncorrected SWD angles. However, it is very difficult to get an ideal dataset from real measurements, containing the relationship between the actual and uncorrected SWD angles under all BWR value conditions. Obtaining training data for BPNN from real measurements is inconvenient, owing to the following reasons. First, inadequate manual operations will cause unpredictable error and induce uncertainty in the training dataset. Second, under low signal-to-noise ratio (SNR) conditions, noise incorporated in the measured data will make it difficult to determine the frequencies of $f_{\text{max}}$ and $f_d$, which results in a deteriorated training dataset. Accordingly, in this paper, we used numerical simulation (Guidi et al. 1995) to simulate Doppler spectra. The $f_{\text{max}}$ and $f_d$ in the simulated Doppler spectra were used to create the required training data for BPNN. This approach provides an efficient and convenient way to generate sufficient information for BPNN training. The validity of applying BPNN for Doppler angle corrections can be examined in the measured data. Comparing the Doppler angles before and after the BPNN corrections, the MAEs of Doppler angles in 10-mm, 4-mm and 1-mm-diameter tubes were 5.73°, 4.08° and 6.16° before corrections, respectively, compared with 1.25°, 1.20° and 1.44° after corrections, respectively, which demonstrated that BPNN algorithm is a feasible way for correcting Doppler angles affected by the BWR effects.

When applying this SWD and BPNN conjoint method for Doppler angle estimations in real measured data, the information of BWR value is needed for BPNN to perform angle correction. Several techniques have been developed for automatically detecting the vessel diameters on 2-D ultrasound images (Newey and Nassiri 2002; Detmer et al. 1990; Gustavsson et al. 1994). With the aid of these techniques, the proposed SWD and BPNN conjoint method becomes feasible to be implemented in clinical ultrasound systems.

The principle of SWD for angle estimation involves the intrinsic spectral broadening (ISB) on the measured Doppler spectrum. The ISB had been numerically and experimentally proven as a phenomenon of spectral broadening in Doppler spectra.

Table 1. The corrected Doppler angles in (a) 10-mm, (b) 4 mm, and (c) 1-mm-diameter tubes

<table>
<thead>
<tr>
<th>Threshold Level</th>
<th>50</th>
<th>55</th>
<th>60</th>
<th>65</th>
<th>70</th>
<th>75</th>
<th>80</th>
<th>MAE</th>
</tr>
</thead>
<tbody>
<tr>
<td>–2 dB</td>
<td>54.5° ± 1.8°</td>
<td>55.6° ± 2.6°</td>
<td>58.3° ± 2.4°</td>
<td>63.5° ± 2.1°</td>
<td>69.9° ± 0.8°</td>
<td>74.9° ± 1.9°</td>
<td>77.9° ± 2.1°</td>
<td>1.51°</td>
</tr>
<tr>
<td>–3 dB</td>
<td>54.9° ± 2.7°</td>
<td>56.4° ± 2.2°</td>
<td>59.1° ± 2.5°</td>
<td>63.8° ± 1.6°</td>
<td>70.3° ± 1.2°</td>
<td>74.8° ± 2.4°</td>
<td>78.7° ± 1.9°</td>
<td>1.46°</td>
</tr>
<tr>
<td>–6 dB</td>
<td>51.7° ± 2.7°</td>
<td>54.8° ± 1.5°</td>
<td>59.1° ± 1.5°</td>
<td>63.6° ± 0.6°</td>
<td>69.2° ± 1.1°</td>
<td>75.1° ± 0.2°</td>
<td>80.4° ± 1.1°</td>
<td>0.79°</td>
</tr>
</tbody>
</table>

Fig. 11. A schematic diagram of an ultrasound beam with moderate width.
ing effect, which originates from the diffraction effect and limited lateral extent of a focused ultrasound beam (Evans et al. 1989; Guidi et al. 2000). ISB presents the received ultrasound signals modulated both in their frequencies and amplitudes. The frequency modulation is caused by a range of insonation angles on a flow line when a focused ultrasound beam is used, which is referred to as geometrical spectral broadening (Newhouse et al. 1987). On the other hand, the amplitude modulation is caused by the finite transit time when a scatterer crosses the ultrasound beam. It had been shown that the bandwidth in the measured Doppler spectrum is inversely proportional to the transit time, which was referred to as transit-time broadening (Ata and Fish 1991). Therefore, the broadened bandwidth in real measurements is a mixture of the geometrical spectral broadening and the transit-time broadening. It has been formulated by Guidi et al. (2000) as

\[ B_0 = \frac{4}{\pi} \sqrt{\ln^2(\gamma) \Delta t + \pi/16 \cdot \Delta f^2} \]

where \( B_0 \) is the spectrum bandwidth, \( \Delta t \) is the transit time of a scatterer crossing the width of ultrasound beam defined by a threshold \( \gamma \) and \( \Delta f \) is the spread of frequencies on Doppler spectra due to phase field curvature along the scatterer trajectory. Because the phase fronts are parallel when the scatterers are crossing the focal zone of the ultrasound beam, the phase modulation is absent, so that \( \Delta f \approx 0 \). The spectral broadening is mainly contributed by transit-time broadening. On the other hand, scatterers crossing the ultrasound beam away from the focal zone will make the spectral broadening mainly to be contributed from geometrical spectral broadening effect, because of the long \( \Delta t \) at those positions out of focus. According to Guidi et al. (2000), the rise and fall of the spectral widths contributed from geometrical spectral broadening and transit-time broadening will cancel out each other and gives an almost constant spectral width in a wide position range near the focal zone. Therefore, the transverse Doppler equations (eqns (A1)-(A4)) can be used to formulate the spectral width contribution of a flow line, whether it is at the focal point or near the focal zone.

In our experiment, the ultrasound probe was immersed in pure water. Ultrasound waves traveled through polyethylene tube, scattered by the scatterers of mimic fluid inside the tube and then through which backscattered to the transducer. Due to difference in sound speeds between the layers ultrasound waves traveled, the acoustic refraction and reflection occurs at boundaries of the water-to-tube and tube-to-fluid interfaces at an oblique incident angle. Figure 12 shows the schematic diagram of the refraction and reflection of an ultrasound beam at an oblique angle. The refraction at water-to-tube boundary can be expressed by Snell’s law as

\[ \sin(\theta_1)/v_{\text{water}} = \sin(\theta_2)/v_{\text{tube}} \]

where \( \theta_1, \theta_2, v_{\text{water}} \) and \( v_{\text{tube}} \) represent the incident angle in water, refraction angle in tube, sound speed in water and sound speed in polyethylene tube, respectively. \( \theta_1 \) and \( \theta_2 \) are the effective Doppler angles for scatterers in mimic fluid and can be rewritten as

\[ \sin(90° - \theta_{\text{water}})/v_{\text{water}} = \sin(90° - \theta_{\text{fluid}})/v_{\text{fluid}} \]

The effective Doppler angle for scatterers can be obtained from

\[ \theta_{\text{fluid}} = \cos^{-1}(\cos(\theta_{\text{water}}) \cdot v_{\text{fluid}}/v_{\text{water}}) \]

in our study, the sound speed of UHDC mimic flow was 1540 m/s, taken from quoted value, and the sound speed was 1480 m/s in water at 20°C. The effective \( \theta_{\text{fluid}} \) after refraction is 48.02° when \( \theta_{\text{water}} \) is 50° and is 79.59° when \( \theta_{\text{water}} \) is 80°. Taking sound speed as 1890 m/s in polyethylene tube, internal reflection only took place when the critical \( \theta_1 \) was larger than 51.5° (\( \theta_{\text{water}} < 38.5° \)) at water-to-tube boundary. Because our measurements were performed in the range of \( \theta_{\text{water}} \) from 50° to 80°, no total reflection occurred in this study. Taking this refraction effect into account, actual Doppler angle should be corrected from the SWD estimated Doppler angle by

\[ \theta_{\text{water}} = \cos^{-1}(v_{\text{water}}/v_{\text{fluid}} \cdot \cos(\theta_{\text{fluid}})) \]

where \( \theta_{\text{water}} \) is the actual Doppler angle and

\[ \theta_{\text{water}} \]
This study aims to propose an improved SWD method for estimating angles in vessels where the velocity gradients might be present in the sample volume. The present study comprises a precalculation step for estimating the uncorrected angle by the SWD method (Lee et al. 1999a, 1999b) and a correction step using BPNN for correcting the effect of VGB on the SWD-calcualted angles. This SWD and BPNN con-joint method takes BWR into consideration and en-ables Doppler angles and flow velocities to be esti-mated in vessels with different diameters, including small vessels. Nevertheless, the proposed method is based on the measurements of \( f_{\text{max}} \) and \( f_\theta \) Doppler spectra. To obtain accurate \( f_{\text{max}} \) and \( f_\theta \), Doppler spectra were averaged over a large number. The requirement of a large number of spectra is mainly owing to the attenuation at the boundaries of polyethylene tube and its surrounding media as a result of the acoustic reflection caused by impedance mismatch. However, this should not be a problem in \textit{in vivo} studies, so that the spectrum number for averaging can be largely reduced. Besides, short-time spectrum estimations using autoregressive model (Yeh and Lee 2002) or cardia-cycle spectrum normalization technique (Lee et al. 1999) might be helpful in the increase of available spectrum number and achievement of better SNR in measured spectra. Apart from this, the Doppler spectra are likely to be affected by complex flow patterns in \textit{in vivo} measurements, \textit{e.g.}, concave flow profile in late systolic phase (Tortoli et al. 1997) and red blood cell aggregation in pulsatile flows (Paeng et al. 2001). Further studies are required for signal improvements in \textit{in vivo} measurements. The proposed method might be a feasible way for estimating Doppler angles in vessels, where Doppler angles are difficult to measure. For example, it could be useful for small vessel measure-ments in upper limb/lower limb, renal glomeruli, umbilical cord, cerebral arterials, \textit{etc}.

\begin{itemize}
  \item \textbf{REFERENCES}
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\end{itemize}


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APPENDIX

APPENDIX A

Conventional Doppler ultrasound assumes an infinitely wide planar wave incident on the scattering particles. The Doppler shift in the measured Doppler spectrum can be expressed as

\[ f_d = \frac{2 \cdot \nu \cdot \cos \theta}{\lambda} \]  

(A.1)

where \( f_d \) is the energy peak frequency in the measured Doppler spectrum (Chabria and Newhouse 1997; Lee et al. 1999a; 1999b; Tortoli et al. 1995), \( \theta \) is the Doppler angle (the beam-to-flow angle), \( \lambda \) is the wavelength of the ultrasound and \( \nu \) is the mean flow velocity.

If a focused ultrasound beam is used in Doppler measurements, the effect of intrinsic spectral broadening (ISB) should be taken into consideration. ISB had been numerically and experimentally proved to be affected by both the diffraction effect (geometrical spectral broadening) and the limited lateral extent (transit-time broadening) of a focused ultrasound beam (Newhouse et al. 1987; Guidi et al. 2000). The ISB at the focus of ultrasound beam has been shown mainly to result from transit-time effect. The bandwidth of measured Doppler spectrum can be represented as

\[ B_d = \frac{2 \nu}{\lambda} \frac{W}{F} \sin \theta \]  

(A.2)

where \( B_d \) is the bandwidth of spectral broadening, \( \theta \) is again the Doppler angle and \( W \) and \( F \) represent the effective diameter and the focal length of a focused ultrasound transducer, respectively.
The maximum frequency, $f_{\text{max}}$, in the measured Doppler spectrum can be derived as the Doppler shift frequency ($f_{d}$) plus one half of the Doppler spectrum bandwidth ($B_{d}/2$) as (Tortoli et al. 1995)

$$f_{\text{max}} = \frac{2v}{\lambda} \cdot \cos \theta + \frac{v}{\lambda F} \cdot \sin \theta,$$  \hspace{1cm} (A.3)  

which is a parameter less affected by low-frequency noise and cutoff frequency of wall-thump filter. Figure A1 illustrates the three parameters ($f_{d}$, $B_{d}$ and $f_{\text{max}}$) on a measured Doppler spectrum. The $f_{d}$ is determined by finding the frequency with maximum power in the Doppler spectrum and $B_{d}$ and $f_{\text{max}}$ are determined by a $-3$-dB threshold, $\gamma_{th}$, in this case.

Once $f_{d}$ and $f_{\text{max}}$ are properly determined, the Doppler angle can be estimated by combining eqns (A1) and (A3), which can be represented as (Lee et al. 1999a)

$$\theta = \tan^{-1} \left[ \frac{f_{\text{max}} - f_{d}}{f_{d}} \cdot \frac{2 \cdot F}{W} \right] = \tan^{-1} \left[ \left( \frac{f_{\text{max}}}{f_{d}} - 1 \right) \cdot \frac{2 \cdot F}{W} \right].$$ \hspace{1cm} (A.4)  

The derivation of the eqn (A4) was considered under large vessel measurement conditions (e.g., BWR $<0.2$). However, in small vessels whose diameters are not much larger than the size of sample volume, the inclusion of velocity gradients poses an influence on the frequencies of $f_{\text{max}}$ and $f_{d}$, which should be corrected in the use of eqn (A4) for Doppler angle estimations.

**APPENDIX B**

The mean absolute error (MAE) of $m$ samples, $x$, is defined as follows,

$$\text{MAE} = \frac{1}{m} \sum_{i=1}^{m} |x_i - T_i|,$$ \hspace{1cm} (B.1)  

where $x_i$ denotes the value of $i_{th}$ sample in $x$ and $T$ is the target value of $x$.  

Fig. A1. An illustration of the three parameters ($f_{d}$, $B_{d}$ and $f_{\text{max}}$) on a measured Doppler spectrum.